2. Trusses and bars in axial load

Contents

Nomenclature

δ	Deformation [mm]	L	Length [mm]
F	Force [N]	ε	Axial strain [mm/mm]
k	Stiffness [N/mm]	ΔT	Temperature change [deg]
σ	Normal stress [N/mm^2]	θ	Oblique angle [rad]
τ	Shear stress [N/mm^2]	α	Thermal expansion coefficient [deg^{-1}]
A	Cross-sectional area [mm^2]	K_t	Stress concentration factor [-]
E	Module of elasticity [N/mm^2]	b,h,r,d	Geometric dimensions [mm]
μ	Coefficient of friction	φ	Pulley angle

2.1 Introduction

Trusses are bars which are assembled with pin-jointed connections and solely loaded in the joints. We know from statics, that trusses act as two force members. This refers to a state of loading that produces pure normal stress which may be tensile or compressive. Hence, trusses are not subjected to shear stress, and can neither transfer bending nor torsion. An example of a simplified truss structure is shown in Figure 2-1. Furthermore, the application of the terms *joints, trusses* and *loads* are shown in order to demonstrate how to use the relevant terminology. Our motivation for learning to master the theory contained in the present chapter is to learn how to analyse this type of structure by calculating the stress and deformation in the separate truss elements.

In the present chapter, the basic formulas for calculation of stresses and deformations of trusses will be introduced. A real-life example of a system dominated by trusses is shown in Figure 2-2. The state of stress in a truss is unidirectional in the sense, that normal stresses only act in one direction, namely the longitudinal (the length coordinate direction), which in the following will be denoted 'the axial direction'. Furthermore, it will be demonstrated how to solve simple statically indeterminate problems.

The introduced theory is valid for trusses of linear elastic materials (meaning that stresses are proportional to strains) in the elastic range. Furthermore, deformations are assumed small to moderate, so the calculated stresses can be assumed acting in the undeformed geometry rather than in the deformed. This enables us not to account for contraction effects when calculating the areas, which serve as basis for calculation of normal stresses.

It is of great importance to note that structural elements act as trusses due to applied boundary conditions (pin-joints) and loading conditions (forces act solely in joints). If truss structures are loaded by significant distributed loads along their lengths or are assembled in a moment-stiff fashion, the separate elements will no longer act as trusses, but as beams. In a similar fashion, trusses with circular cross-sections will act as shafts if fixated or assembled by torsion-stiff connections.

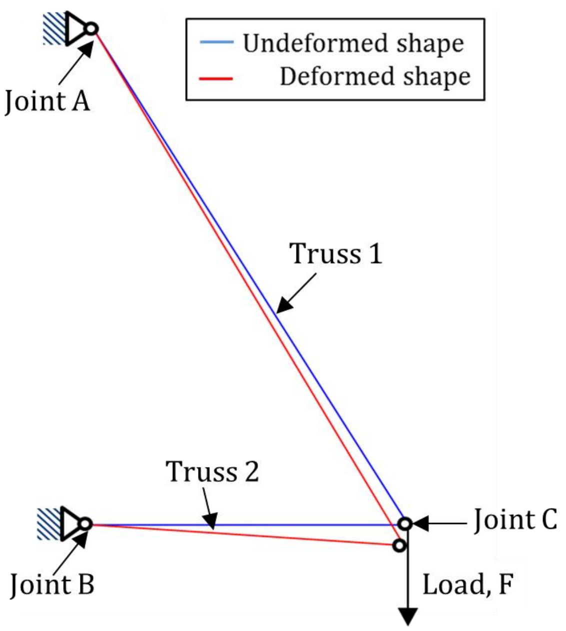


Figure 2-1 Example of an idealized console truss structure

Figure 2-2 Typical truss dominated structure, the Düsseldorf Airport terminal building (Photo taken by author)

2.2 Physics re-cap: spring mechanics

Initially, two very useful formulas will be repeated (or introduced) though they may seem lightly off-topic for the reader. We shall consider the problem of replacing the stiffness of a number of linear springs with a single spring stiffness acting in the same manner as the larger system. This will be referred to as an equivalent spring stiffness. By linear spring stiffness, we simply refer to a spring for which the force-deformation (F-δ) characteristics is linear, i.e. forms a straight line with constant slope equal to the stiffness *k,* see Figure 2-3-A.
Initially, considering Figure 2-3-B, two springs in series are shown. The deformation of the entire system can be observed to be given by the sum of deformations of both springs. Hence, the following must hold

(Springs in series)

$$\delta = \delta_1 + \delta_2 = \frac{F}{k_1} + \frac{F}{k_2} = F\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$
$$\text{so } \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \qquad (2\text{-}1)$$

In which k is the equivalent spring stiffness of the two springs. It is noted that the force is the same in both springs.
Now considering two springs mounted in parallel as shown in Figure 2-3-C, the total force on the system must due to force equilibrium equal the sum of forces generated respectively by the two springs. On equation form, it follows that

(Springs in parallel)

$$F = F_1 + F_2 = k_1\delta + k_2\delta = (k_1 + k_2)\delta$$
$$\text{so } k = k_1 + k_2 \qquad (2\text{-}2)$$

In which the deformation of both springs is equal.
This enables us to determine equivalent stiffness measures for larger more complex spring systems, as long as these are linear. The two equivalent stiffness parameters can on mathematical form be observed to function opposite electrical resistances connected respectively in series and in parallel.

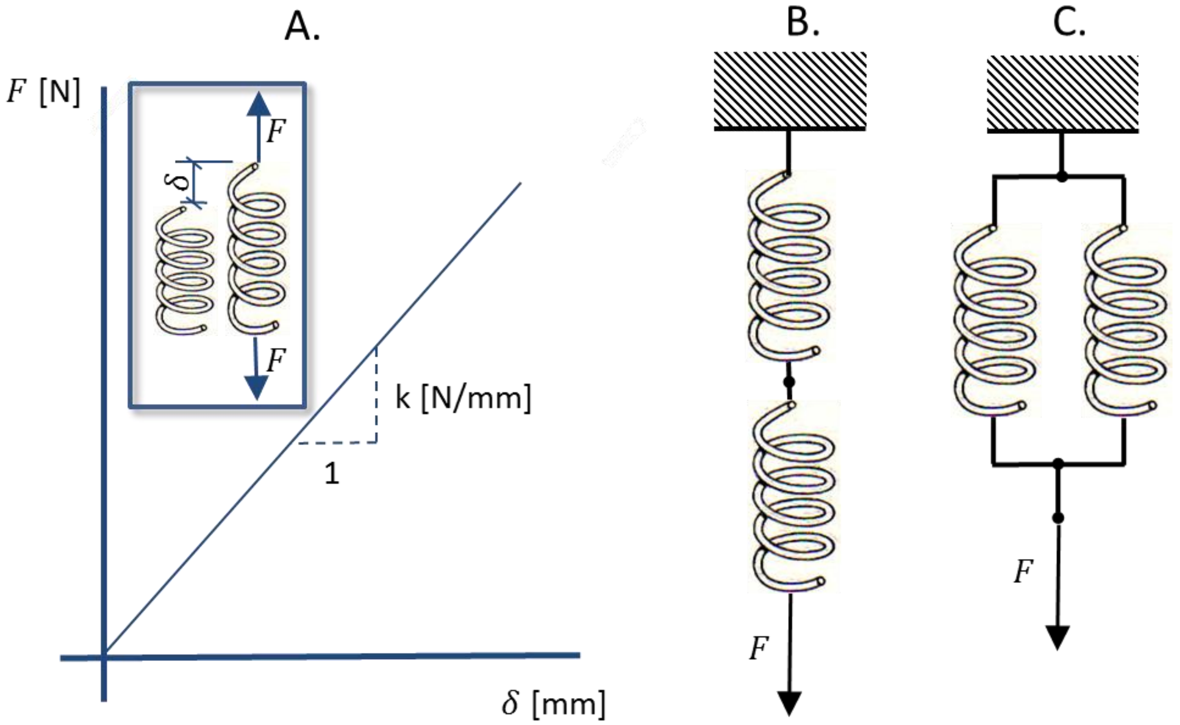


Figure 2-3 A: Linear spring force-deformation relation, B: linear springs coupled in series, C: linear springs coupled in parallel

2.3 Statics: Equilibrium of internal and external forces in a section

Before proceeding to methods for determination of stresses and deformations, we recall that calculations of these quantities must be based on internal rather than external forces. The procedure required for calculation of sectional forces is illustrated in Figure 2-4.The following does not apply for beams in bending in unmodified form. In general, the following rules must be kept in mind:

- Calculation of stresses is based on internal (also called sectional) forces and moments. The internal forces are determined by introducing a fictive cut (or section) in the structure through the part where stresses are required determined.
- The forces acting in the cut must maintain equilibrium with the external forces – if equilibrium was not maintained, the structure would accelerate. You may consider equilibrium of the segment left or right to the cut - the result is the same
- You may add the internal forces in the section you are considering pointing in a direction of your own choice. If the physical direction of the force is opposite what you assumed, the force or moment will come out negative. However, it is often easiest to add the force in the direction you think it will work in reality
- When cutting a structure in a given section you maintain equilibrium with externally applied loads but not forces from other sections. If you have cut multiple sections through a structure, the forces in these shall not be considered and drawn when considering the sectional forces in other cuts. You only consider one section at the time along with the external loads
- For members in 1D loading (like axial force and torsion) which are not subjected to distributed loads or torques, the internal force and moment components can only vary

in points where external loads are added. Hence, the values of the internal forces can be taken as constant between these points

Now considering the bar AD shown in Figure 2-4-I subjected to two loads, F_B and F_D, the reaction in A can be determined on basis of the free body diagram in Figure 2-4-II

$$\begin{aligned} &\sum F = 0 \\ &\rightarrow -F_A + F_B + F_D = 0 \\ &\rightarrow \quad F_A = F_B + F_D \end{aligned} \tag{2-3}$$

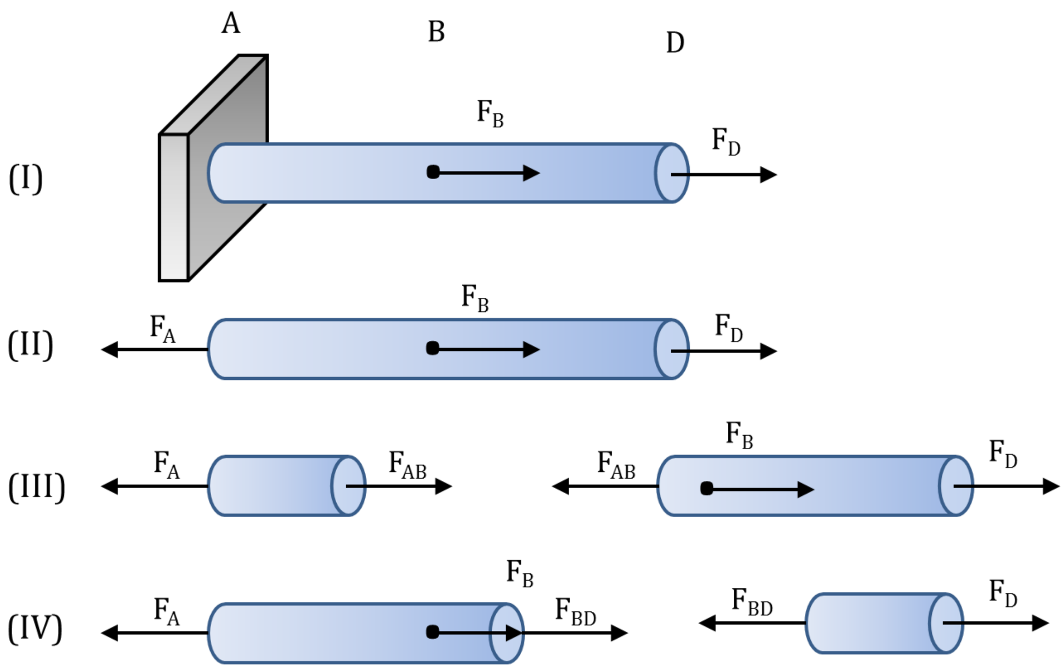


Figure 2-4 Calculation of internal forces in the cylinder AD subject to two axial loads F_B and F_D

The internal force will due to the two external loads vary in the two segments AB and BD. Considering the section in Figure 2-4-III, the internal force is calculated by

$$\begin{aligned} &\text{Left segment: } \sum F = 0 && \text{Right segment: } \sum F = 0 \\ &\rightarrow F_{AB} - F_A = 0 && \rightarrow -F_{AB} + F_B + F_D = 0 \\ &\rightarrow F_{AB} = F_A = F_B + F_D && \rightarrow F_{AB} = F_B + F_D \end{aligned} \tag{2-4}$$

Equivalently, the internal force in the section in Figure 2-4-IV is calculated by

$$\begin{aligned} &\text{Left segment: } \sum F = 0 && \text{Right segment: } \sum F = 0 \\ &\rightarrow F_{BD} + F_B - F_A = 0 && \rightarrow -F_{BD} + F_D = 0 \\ &\rightarrow F_{BD} = F_A - F_B = F_D && \rightarrow \ F_{BD} = F_D \end{aligned} \tag{2-5}$$

It is observed, that the internal forces may be calculated using either the left or right segment of the structure. Both equations are obtained by requiring internal and external forces to be in equilibrium. In both sections, the internal forces are calculated as being positive, meaning that these are directed as assumed (see Figure 2-4).

2.4 Mode of deformation and strain relation

The axial strain ε_x in a bar is defined as the axial deformation δ divided by the length L. The deformations are assumed small, which we usually write δ<<L (meaning δ is a lot smaller than L) so we may neglect the change of length due to elongation and apply the original length in the undeformed state as measure for L. We have

$$\varepsilon_x = \frac{\delta}{L} \tag{2-6}$$

If the bar is made of a linear elastic material, Hook's law for uni-axial stress may be applied, see Figure 2-5.We get the uni-axial stress σ_x by the expression

$$\sigma_x = E\varepsilon_x = E\frac{\delta}{L} \tag{2-7}$$

If a bar is constituted by multiple sections with different lengths L, cross-sectional areas A and modules of elasticity E, the total deformation δ is obtained by the equation

$$\delta = \sum_{i=1}^{n} \frac{F_i L_i}{E_i A_i} \tag{2-8}$$

in which F is the internal force in the *i'th* segment of the bar. In example 2A it is demonstrated how this equation is applied for analysis.

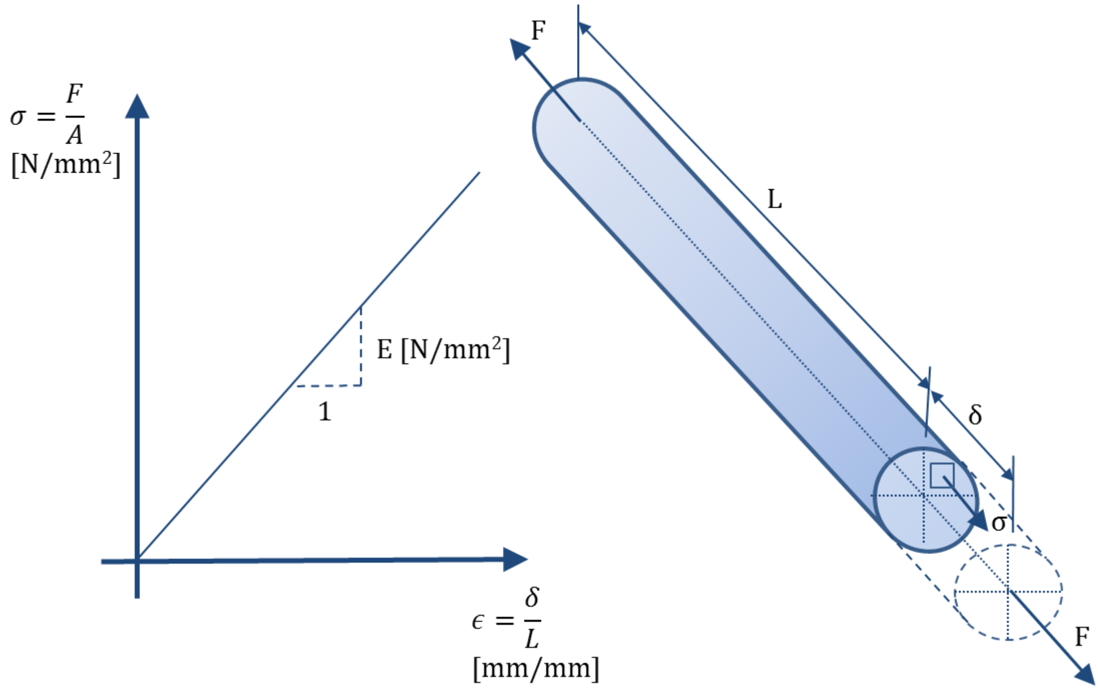


Figure 2-5 Hook's law for axial loads (linear constitutive relation for uni-directional stress) for stress-strain calculations

2.5 Stress calculation

The derived stress expression in Eq. 2-8 must comply with the definition of relation between stress and internal axial force obtained in chapter 1. This was given by

$$F_x = \int \sigma_x dA \tag{2-9}$$

If the obtained stress expression is substituted into the equation above, the following is obtained for a constant value under the integral symbol

$$F_x = \int E\frac{\delta}{L}dA = EA\frac{\delta}{L} \rightarrow \frac{F}{A} = E\frac{\delta}{L} \tag{2-10}$$

In general terms, we write

$$\sigma = \frac{F}{A} \tag{2-11}$$

While this might seem to be an overkill in the present context, this will be the approach to be applied when deriving more complex stress expression for bending and torsion.

2.6 General strategies: axial loads, torsion and bending

The following general strategy can now be formulated. This is of great importance since we in the following chapters will apply this when analyzing shafts in torsion (ch. 3) and beams in pure bending (ch. 4).

Derivation strategy for basic stress and deformation problems

1. Consider the **static equilibrium between internal and external forces** in the analyzed section
   In this case $\sum F = 0$ for the considered section
2. Since the distribution of stress at this state is statically indeterminate, the **state of deformation** caused by the internal forces is considered and applied as basis for derivations of a **strain relation**
   For axially loaded members we have $\varepsilon_x = \frac{\delta}{L}$
   Think of a strain as a 'unit deformation', hence, a deformation that is normed with respect to the total dimensions of an elastic body and remember, that the content of this course only is valid for small strains (we write $\varepsilon_x \ll 1$)
3. A linear elastic material law is applied (as **constitutive relation**) allowing conversion of strains to stresses on infinitesimal form.
   In the present case, $\sigma_x = E\frac{\delta}{L}$
   Remember to think of a stress as a force divided by an area. Forces normal to a section cause normal stresses σ and forces parallel to a section shear stresses τ.
4. The obtained infinitesimal stress expression is integrated over the cross-section and hereby related directly to the internal forces.
   Since the stress for an axially loaded member is uniformly distributed, $F_x = EA\frac{\delta}{L} = \sigma_x A$

Having completed an analysis like this enables us to establish a toolbox for standard problem solving for engineering analysis. The strategy is simple and goes like this:

Standard problem solving strategy for basic stress and deformation problems

1. Formulate the equilibrium equations between internal and external forces in all relevant sections.
2. Knowing the internal forces, apply the derived 'ready-to-use' formulas to calculate stresses in all relevant sections with internal forces and cross-sectional parameters as input
   In the present context, $\sigma = \frac{F}{A}$
3. Knowing the internal forces, apply the derived 'ready-to-use' formulas to calculate the deformations of all relevant sections with internal forces, cross-sectional parameters and material parameters as input
   In this case, $\delta = \sum \frac{FL}{EA}$

We are going to do a calculated example, to see how this works when doing engineering analysis.

Calculated example 2A: Compound cylinder with axial loads

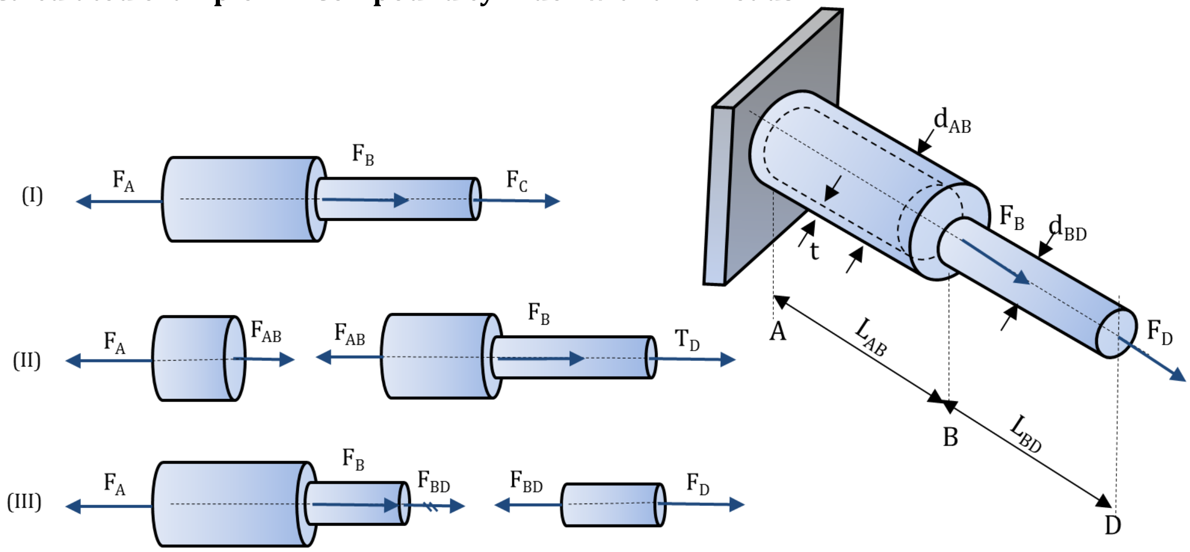


Figure 2-6

A compound cylinder is subjected to loads F_B=200 kN and F_D=100. The geometry is given by the parameter set t=10 mm, $d_{AB,O}$=125 mm, $d_{AB,I}$=105 mm, d_{BD}=80 mm, L_{AB}=300 mm and L_{BD}=200mm. The segment AB is made of aluminum with module of elasticity $E_{AB}=70{\cdot}10^3$ MPa and the segment BC of steel with module of elasticity $E_{BD}=210{\cdot}10^3$ MPa. The short cap holding the two segments together can be neglected. Calculate **a)** the normal stresses in the two segments AB and BD, **b)** the deformation of point C.

Solution:

Step 1: The internal forces are determined from the free-body diagrams shown in Figure 2-6. For segment AB we obtain the following equation on basis of the right segment shown in Figure 2-6-III

$\sum F = 0 \rightarrow -F_{AB} + F_B + F_D = 0 \rightarrow F_{AB} = F_B + F_D$

In a similar fashion, F_{BD} is obtained on basis of the right segment in Figure 2-6-IV

$\sum F = 0 \rightarrow -F_{BD} + F_D = 0 \rightarrow F_{BD} = F_D$

Step 2: The normal stresses can now be calculated by

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{F_B+F_C}{\frac{\pi}{4}\left((d_{AB,o})^2-(d_{AB.i})^2\right)} = \frac{(100+200)\cdot 10^3 N}{\frac{\pi}{4}((125\,mm)^2-(105\,mm)^2)} = 83.0\frac{N}{mm^2}$$

$$\sigma_{BD} = \frac{F_{BD}}{A_{BD}} = \frac{F_D}{\frac{\pi}{4}d_{BD}} = \frac{100\cdot 10^3 N}{\frac{\pi}{4}(80mm)^2} = 19.9\frac{N}{mm^2}$$

Step 3: The deformation of point C can be calculated as the sum of deformations of the two segments AB and BD:

$$\delta = \sum\frac{FL}{EA} = \frac{F_{AB}L_{AB}}{E_{AB}A_{AB}} + \frac{F_{BD}L_{BC}}{E_{BC}A_{BC}} = \frac{(F_B+F_D)L_{AB}}{E_{AB}\left(\frac{\pi}{4}\left((d_{AB,o})^2-(d_{AB.i})^2\right)\right)} + \frac{F_C L_{BD}}{E_{BD}\frac{\pi}{4}(d_{BD})^2} +$$

$$= \frac{\left((200+100)\cdot 10^3 N\right)300mm}{70\cdot 10^3\frac{N}{mm^2}\left(\frac{\pi}{4}((125\,mm)^2-(105\,mm)^2)\right)} + \frac{(100\cdot 10^3 N)200\,mm}{210\cdot 10^3\frac{N}{mm^2}\frac{\pi}{4}(80mm)^2}$$

$$= 0.356mm + 0.0189mm = 0.375mm$$

2.7 Statically indeterminate problems

For bars in axial loads, only one equation of equilibrium is available to solve for the reactions. However, if constraints are added in a fashion producing more than one reaction to external loads, the considered problem becomes statically indeterminate. The most common case is shown in Figure 2-7. A bar is constrained between two walls and an external load is applied. Obviously, an additional equation is required to solve for the wall reactions, since two equations are needed to solve for two unknowns.

The common strategy for linear elastic problems is to apply the principle of superposition. We will split the problem into two separate load cases: one considering only the external load F_B and the caused deformation δ_F and a second case considering only the right reaction force R_D and the corresponding deformation $\delta_{R.}$ These deformations can be calculated by application of the equations in section 2.4. However, if added, these deformations have to add up to zero to form the original load case

$$\delta_F + \delta_R = 0 \tag{2-12}$$

This type of equations are called *equations of compability* and are required to solve statically indeterminate problems. Application of this principle is illustrated in calculated example 2B.

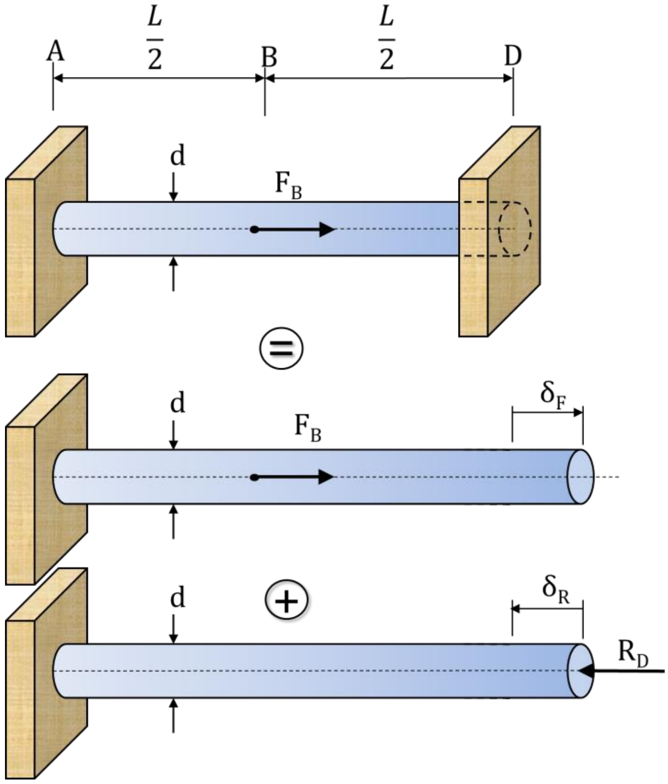


Figure 2-7 A bar constrained between two walls

2.7.1 Thermal stresses in bars

We shall now consider the simplest case of thermal stresses imaginable, namely, a bar which is mounted in a manner, so it is constrained against elongation. In Figure 2-8, this is visualized as two walls. The bar will be considered free of residual stresses, which in this context means, that the bar is free of stress. However, if the temperature is increased with ΔT, the bar would elongate if unconstrained. Since the wall prevents elongation due to thermal loads, a compressive reaction causing a compressive stress is introduced in the bar. We will assume that the bar cannot deform sideways, hence, in the transverse direction.

The considered problem is statically indeterminate. We recall from statics, that this means that the number of reactions we wish to calculate is larger than the number of equilibrium equations available. For statically indeterminate problems, the state of deformation must be considered along with the equilibrium in order to calculate the acting forces. The problem can again be solved by superposition. This solution technique refers to, that the problem is split into separate load cases, and the deformation is obtained as the sum of deformations caused by the separate loads.

As shown in Figure 2-8, we will consider two load cases: a) the elongation of the bar due to the increase of temperature (II) and the corresponding compression of the bar induced by the wall constraint (III).

For (II), we recall from physics that the deformation due to thermal expansion is given in terms of the thermal expansion coefficient α, ΔT and the length L by

$$\delta_T = \alpha \Delta T L \tag{2-13}$$

On the other hand, the compressive deformation due to the wall reaction for (III) is in accordance with Eq. 2-9 given by

$$\delta_P = \frac{PL}{EA} \tag{2-14}$$

The deformation of the lowest point must however add up to 0. We obtain the following expression

$$\begin{aligned} &\delta_T + \delta_P = 0 \\ &\rightarrow \frac{PL}{EA} + \alpha \Delta T L = 0 \\ &\rightarrow P = -\alpha \Delta T\, EA \end{aligned} \tag{2-15}$$

If the stresses are required, these are easily obtained since the bar is in a state of uni-axial stress. Hence,

$$\sigma = \frac{P}{A} = -\alpha \Delta T E \tag{2-16}$$

The compressive load induced by thermal expansion may cause the considered bar to buckle sideways. The calculation procedure for design against buckling will be introduced in chapter 9.

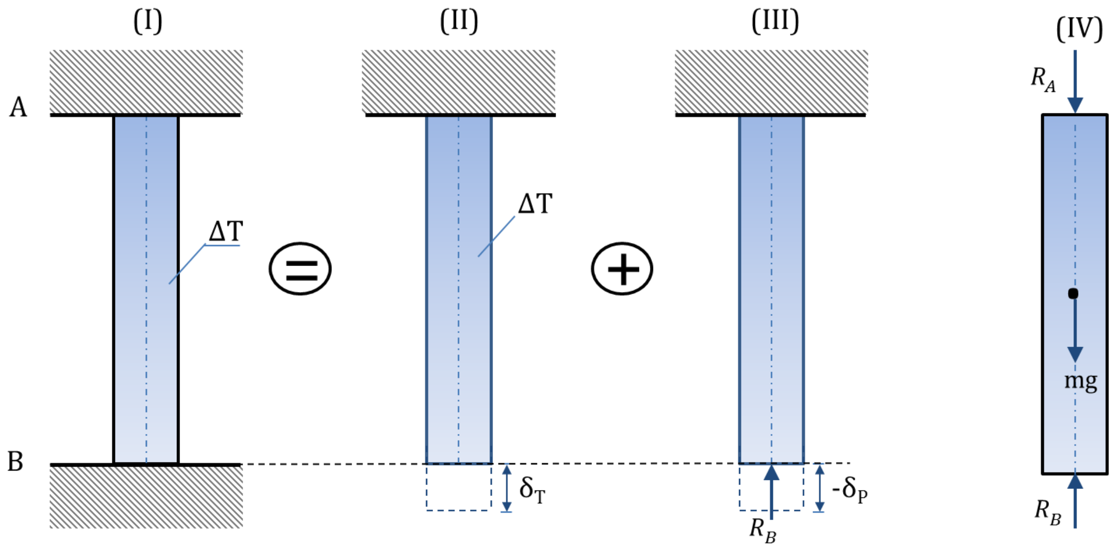


Figure 2-8 Solution of statically indeterminate thermal stress problems by superposition

Calculated example 2B: Axially loaded bar constrained between two walls

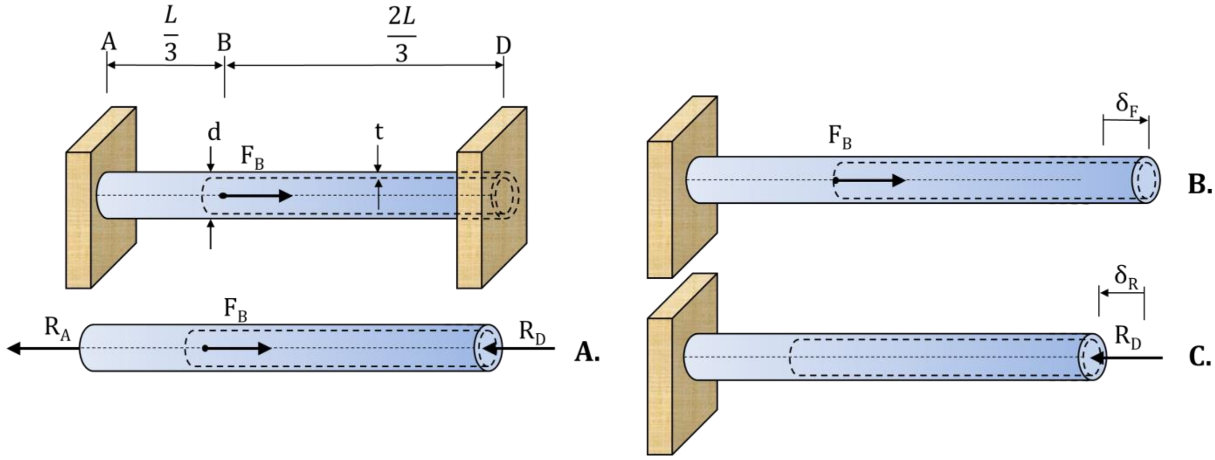


Figure 2-9

A pipe of length L=2 m is constrained between two walls while a load F_B=55 kN is applied in point B. The section AB is solid with a diameter d=110 mm, while the section BD is hollow with wall thickness t=20 mm. The pipe is made of aluminum with elastic modulus E=$70 \cdot 10^3 N/mm^2$. Calculate **a)** the reaction forces in A and D, and **b)** the stresses in the sections AB and BD

Solution:

From the free-body diagram (Figure 2-9-A), the following equation of equilibrium is obtained

$\sum F = F_B - R_A - R_D = 0$

However, we cannot solve this for the reactions, since the equation has two unknowns. The system is therefore statically indeterminate. We therefore consider the two load cases shown in Figure 2-9-B and Figure 2-9-C. These represent respectively the external load and the reaction at D along with their corresponding deformations. On this basis, the equation of compability, $\delta_F + \delta_R = 0$, is obtained. In order to calculate the required deformations, we will need the cross sectional areas. These are given by

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(110\,mm)^2 = 9503.3\,mm^2$$

$$A_{BD} = \frac{\pi}{4}(d_{BD}^2 - (d_{BD} - 2t)^2) = \frac{\pi}{4}\left((110\,mm)^2 - \left(110\,mm - 2(20\,mm)\right)^2\right) = 5654.9\,mm^2$$

<u>Considering only the external load</u> and the corresponding deformation (Figure 2-9-B), we observe that the internal forces in the two sections *AB* and *BD* are given by $F_{AB}=F_B$ and $F_{BD}=0$. The deformation δ_F can now be calculated

$$\delta_F = \sum \frac{FL}{EA} = \frac{F_{AB}\left(\frac{L}{3}\right)}{EA_{AB}} + \frac{0 \cdot \left(\frac{2L}{3}\right)}{EA_{AB}} = \frac{55 \cdot 10^3 N \left(\frac{2000\,mm}{3}\right)}{70 \cdot 10^3 \frac{N}{mm^2} \cdot 9503.3\,mm^2} = 0.055\,mm$$

<u>Now, considering only the reaction force in D,</u> the internal forces are constant and we have $F_{AB}=F_{BD}=R_D$. Treating R_D as unknown, the deformation due to the reaction is given by

$$\delta_{R_D} = -\sum \frac{FL}{EA} = -\frac{R_D}{E}\left(\frac{\frac{L}{3}}{A_{AB}} + \frac{\frac{2L}{3}}{A_{AB}}\right) = -\frac{R_D}{70 \cdot 10^3 \frac{N}{mm^2}}\left(\frac{\frac{2000\,mm}{3}}{9503.3\,mm^2} + \frac{\frac{2 \cdot 2000\,mm}{3}}{5654.9\,mm^2}\right)$$

$$= -4.37 \cdot 10^{-6} \frac{mm}{N} R_D$$

Substituting this into the equation of compability, we obtain one equation with one unknown

$$4.37 \cdot 10^{-6} \frac{mm}{N} R_D = 0.055\,mm \rightarrow R_D = \frac{0.055\,mm}{4.37 \cdot 10^{-6} \frac{mm}{N}} = 12.6 \cdot 10^3 N$$

Having solved for R_D, we return to the free-body diagram. We have the internal forces $F_{AB}= R_A$ and $F_{BD}=R_D$. Using the equation of equilibrium, we may now solve for the last reaction

$$R_A = F_B - R_D = 55.0 \cdot 10^3 N - 12.6 \cdot 10^3 N = 42.4 \cdot 10^3 N$$

The stresses can now be calculated

$$\sigma_{AB} = \frac{R_A}{A_{AB}} = 4.46 \frac{N}{mm^2} \qquad \sigma_{BD} = \frac{-R_D}{A_{BD}} = -2.23 \frac{N}{mm^2}$$

In example 2B, we notice, that if the cylinder had the same cross sectional area all the way through, the equation of compability after substitution of the deflections in terms of reactions is reduced to

$$R_D = \frac{P}{3} \qquad R_A = P - R_D = \frac{2P}{3}$$

2.7.2 Stress concentrations in axial loads

This far, only stresses in bars and trusses in sections where stresses are evenly distributed have been considered. However, in details containing notches, a local increase of stress will occur. Notches are geometrical discontinuities which cause a concentration of stress. Typical examples of notches are changes of diameters, holes and sharp corners. An example of a notch is shown in Figure 2-10 in section B containing the fillet radius r in the transition between a large and a small diameter in a compound cylinder. The analytical procedure for calculation of the exact stresses as solution to differential equations formulated on basis of the general theory of elasticity is beyond the scope of a basic course in strength of materials. However, analytical solutions have been mapped in form of standard solutions and visualized in appropriate diagram form, see Figure 2-11. In general, a stress concentration factor denoted K_t is obtained on basis of the geometry of the considered design. Once the stress concentration factor is available, the maximum stress in the considered notch can be obtained by

$$\sigma_{notch,max} = K_t \sigma_{nom} \tag{2-17}$$

In this equation, the nominal stress, σ_{nom} is given by F_D/A_D. It is of great importance to emphasize, that the stress concentrations only are valid for a given geometry and a given type of load. If the considered compound cylinders were subjected to bending or torsion, the stress concentration factor would change. Therefore, a different curve would have to be applied in order to calculate the correct stress concentration factors. Curves for a high number of different standard geometries and various loads are available in the technical literature.

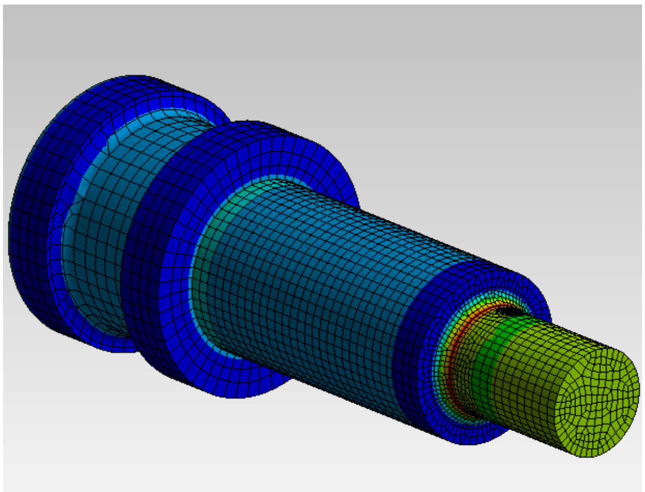


Figure 2-10 Stress concentrations calculated in notches using numerical methods (finite element analysis)

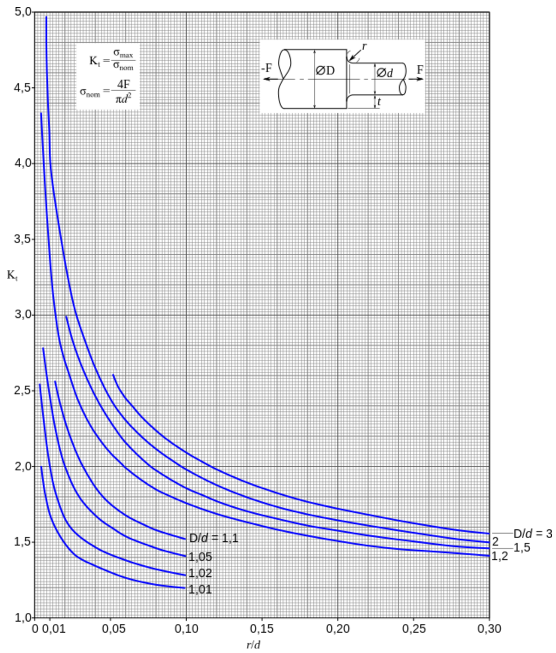


Figure 2-11 Example of stress concentration factor diagram, in the present case for a compound cylinder (from Wikimedia)

2.8 The mechanics of ropes and bands on pulleys

The general mechanical equilibrium of wires and ropes with neglectable bending stiffness leads to the hyperbolic catenary equations from statics[1]. However, this far the problem related to the forces in a wire rope tightly fit around a pulley or capstan has not been considered, see Figure 2-12. In general, if a tensile line pull F is added as load, the force required to hold this load F_{hold} on the other side of the pulley is smaller than F, since some of the load is carried by the friction μ acting along the pulley. This effect is accounted for by the following expression usually referred to as the Eytelwein or capstan equation

$$F = F_{hold} e^{\varphi\mu} \tag{2-18}$$

The holding force can be observed to decay exponentially with the friction and the number of turns a wire is wound around a pulley. The derivation requires determination of the normal force acting on an infinitesimal segment of wire and is in the present context beyond our scope[2]. Equation (2-17) is often needed in mechanical design.

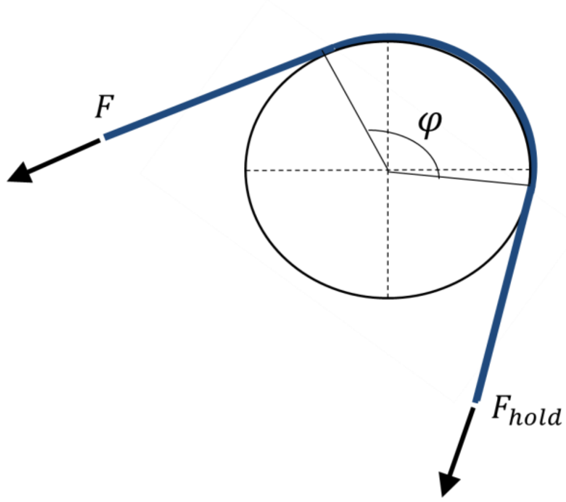


Figure 2-12 A wire with no bending stiffness on a capstan

[1] Online notes on the derivation of the catenary equations
[2] MIT-courseware on the derivation of the capstan equation

Historical example: stress concentration factors, the de Havilland Comet-1

The comet-1 airliner, was the first series of commercial jetliners, see Figure 2-13. The first planes went into service in 1952, but the planes of the Comet model suffered three fatal accidents in 1953-54, before all planes were grounded and crash investigations were initiated. The crash-commission identified multiple design faults, but pointed out that that stress concentrations due to insufficient fillets in the window sections had reduced the service life time in fatigue. This had led to catastrophic failure by rupture of the plane cabins. A sketch of the comet window sections are shown in Figure 2-15.

(Fotos from Wikimedia)

Figure 2-13 The comet-1 jetliner

Figure 2-14 Comet airliner cocooned and stored in Heathrow airport after grounding of all planes of this model in 1954

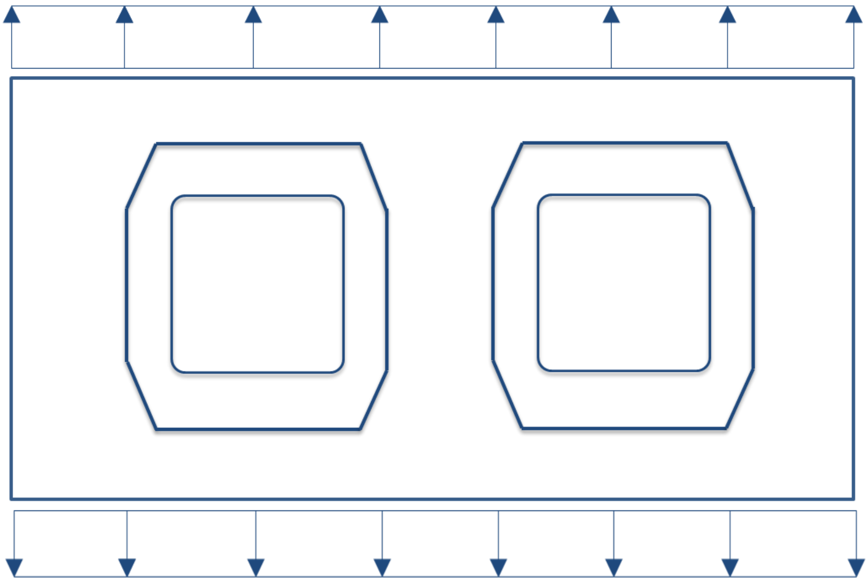


Figure 2-15 The window section of a Comet-1 jetliner. Note the sharp window edges. Where would a fatigue crack form in this section and how would it develop?

Example of truss mechanics: The Bridgebuilder computer game

The Bridgebuilder computer game (download here) is an excellent example of truss mechanics. This epic computer game classic which is more additive and sustains a higher awesomeness factor than all GTA-versions is all about constructing a bridge of trusses. Each level is passed when a train has crossed the bridge without this leading to structural collapse, see Figure 2-16. The game offers an excellent opportunity for training your intuition in structural mechanics.

Bridgebuilder is based on a very simple finite element code, corresponding to a matrix formulation of the mechanics described in this chapter. A Matlab example from a similar code is shown in Figure 2-16. This code should have been made available on Moodle, but if your Professor forgets about him, remind him that you want it[3].

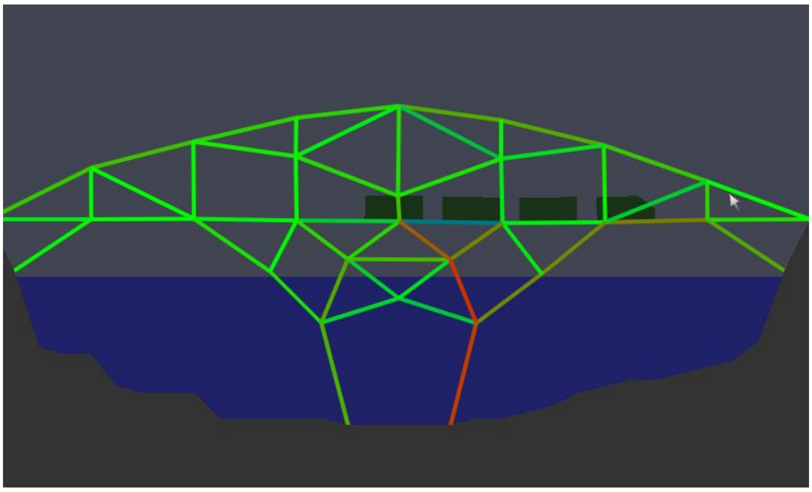


Figure 2-16 The Bridgebuilder computer game

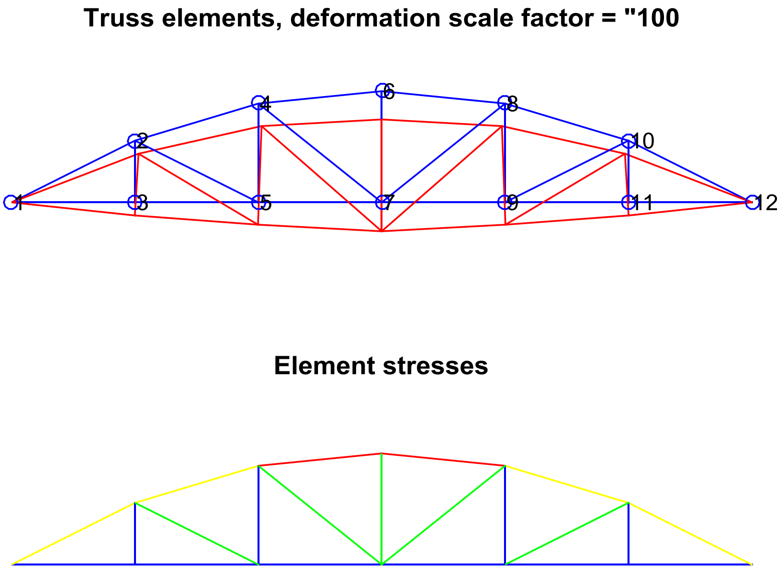


Figure 2-17 Results from a Matlab code reconstructing the Bridgebuilder physics engine

[3] The essential theory related to conversion of the truss equations to matrix form is available in this online note

Problems

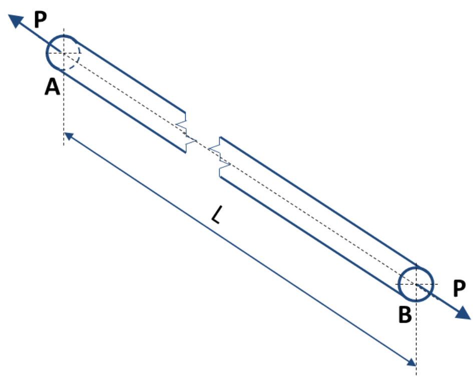


Figure P2.1

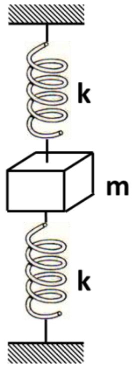


Figure P2.2

Problem 2.1

A straight wire rope of length L=0.5 km is loaded axially by a force P=120 kN and has elastic modulus E=150 GPa. If the maximum allowable normal stress is given by 100 N/mm^2, **a)** calculate the required diameter based on the allowable normal stress, **b)** calculate the required diameter of the wire rope, if the maximum allowable axial strain is 0.5%.

Ans.: **a)** d=39.1 mm, **b)** 14.3 mm

Problem 2.2

Determine the total system stiffness of the spring coupling shown in Figure 2.2

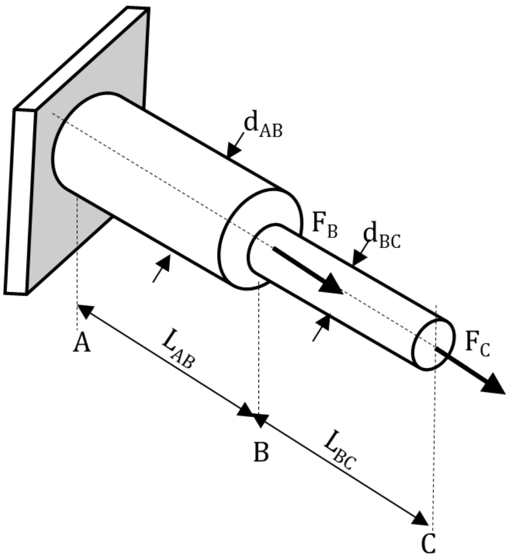


Figure P2.3

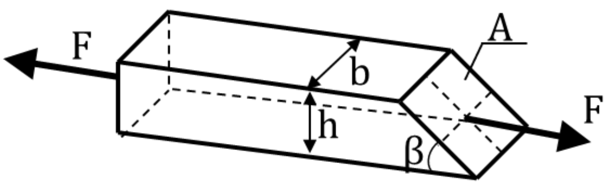


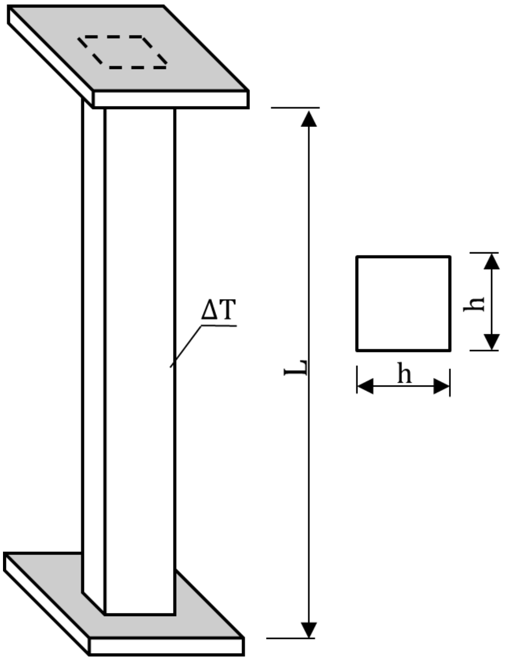
Figure P2.4

Problem 2.3

Figure P2.3 shows a compound solid cylinder with diameters and lengths d_{AB}=100 mm, d_{BC}=75 mm, L_{AB}=500 mm and L_{BC}=600 mm. The cylinder is loaded in the axial direction as shown on the figure with the loads $F_B=100\cdot10^3$N and $F_C=200\cdot10^3$N. The cylinder is made of aluminum with elastic modulus $E=70\cdot10^3$N/mm^2. **a)** calculate the reaction force at point A, **b)** calculate the stresses in the two segments AB and BC, **c)** calculate the axial deformation of point C.

Ans: a) $F_A=300\cdot10^3$N, b) σ_A=38.2 N/mm^2, σ_B=45.3 N/mm^2, c) δ_C=0.661 mm

Problem 2.4
In Figure P2.4, a solid bar with rectangular cross-section and an oblique section is shown. The bar is subjected to an axial load F=200 kN in each end. For h=50 mm, b=70 mm and the angle β=60 deg. calculate **a)** the normal stress in the oblique plane with area A, **b)** the shear stress in the oblique plane with area A, **c)** for which oblique angles will the normal and shear stress have maximum values?
Ans: a) σ=42.9 N/mm^2, b) τ=24.7 N/mm^2



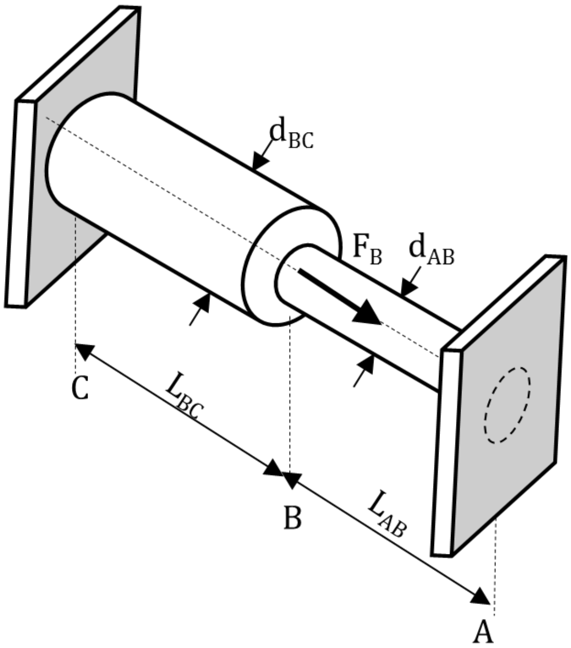


Figure P2.5 [In particular not true to scale] **Figure P2.6**

Problem 2.5
The solid bar shown in Figure P2.5 has quadratic cross-section with side length h=25 mm. The bar is fixated between two walls, so it cannot elongate. The material, which the bar is made of, is structural steel with elastic modulus E=210 GPa and thermal expansion coefficient $\alpha=12\cdot10^{-6}$ deg^{-1}.
The bar is in the initial state free of stress. Calculate **a)** the normal force in the bar if the temperature of the bar is increased with ΔT=25 deg, **b)** the normal stress in the bar.
The length of the bar is given by L=2 m.
Ans: a) P=-39.4 kN, b) σ=-63 N/mm^2

Problem 2.6
The compound cylinder shown in Figure P2.6 has geometry given by d_{AB}=50 mm, L_{AB}=600 mm, d_{BC}=75 mm, L_{BC}=500 mm. Segment AB is made of brass with module of elasticity E_{AB}=110 GPa and thermal expansion coefficient $\alpha_{AB}=19\cdot10^{-6}$ deg^{-1} while segment BC is made of aluminum with module of elasticity E_{BC}=69 GPa GPa and thermal expansion coefficient $\alpha_{BC}=23.1\cdot10^{-6}$ deg^{-1}. The bar is in the initial state free of stress. Calculate the normal stress in the two sections of the bar if **a)** an external load $F_B = 100\,kN$ is applied in B, **b)** a uniform temperature increase of ΔT=35 deg is applied, **c)** if both the external load from a) and the temperature increase from b) is applied
Ans: a) σ_{AB}=-18.9 N/mm^2, σ_{BC} =14.2 N/mm^2, b) σ_{AB}=-92.6 N/mm^2, σ_{BC} =-41.2 N/mm^2, c) σ_{AB}=-111.5 N/mm^2, σ_{BC} =-26.9 N/mm^2

Problem 2.7

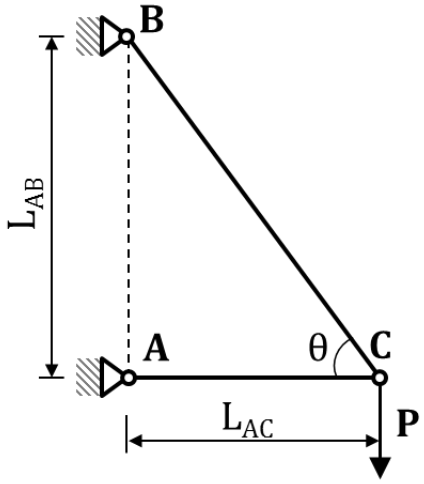


Figure 2.7

For the truss system shown in Figure P2.7, for P=100kN, L_{AC}=2m and θ=60 deg. **a)** Calculate the reaction forces in point A and B.
The trusses are made of rectangular solid steel bars with cross-sectional dimensions h×b=100 mm × 50 mm. **b)** Calculate the stresses in the bars AC and BC.

Ans.: **a)** $R_{A,x}$=58 kN, $R_{A,y}$=0
$R_{B,x}$=-58 kN, $R_{B,y}$=100 kN
b) σ_{AC}=-12N/mm²,σ_{BC}=23 N/mm²

Problem 2.8

Two pieces of wood are assembled with splices glued to the surfaces of contact. A force of magnitude P=25kN is applied to the assembly. A gap g=10 mm is required between the members which are b=150 mm wide. If the allowable shear stress in the glue is given by $\tau_{all} = 3\frac{N}{mm^2}$, calculate the minimum required length L.

Ans.: L=65.56 mm

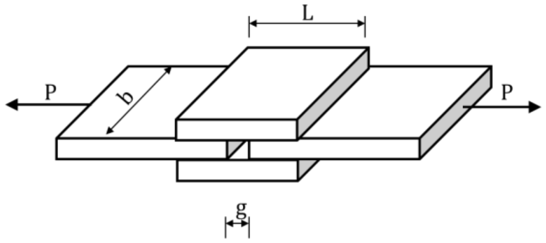


Figure 2.8

Problem 2.9

The frame shown in Figure P2.9 is loaded by a vertical force P=245 kN in point B. The cross sectional areas are 2500 mm² and 1800 mm² for trusses AB and AD. All trusses are made of structural steel with elastic modulus E=210 GPa. The frame is of height h=3 m and length l=10 m.
For all members in the frame, determine a) the internal forces, b) the deformations, c) the normal stresses

Ans.:
a) F_{AB}=-238.1 kN, F_{AD}=204.2 kN
b) δ_{AB}=-2.64 mm, δ_{AD}=2.70 mm
c) σ_{AB}=-95.24 N/mm²,σ_{AD}=113.4 N/mm²

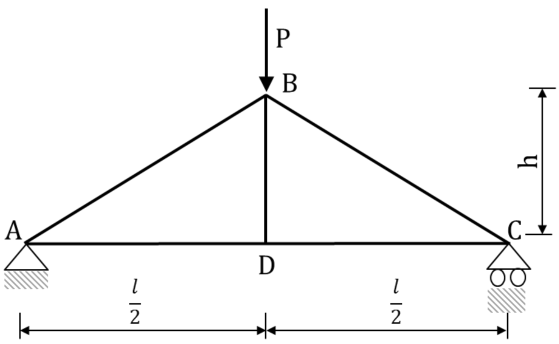


Figure P2.9

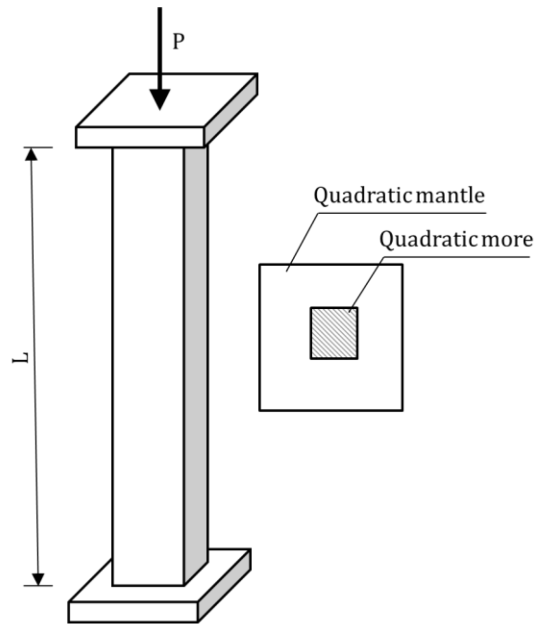


Figure P2.10

Problem 2.10
A composite bar of length L=200 mm is constituted by an outer aluminum mantle with side lengths of 60 mm and a steel core with side lengths of 20 mm. Both mantle and core has quadratic cross sections. The module of elasticity of aluminum is given by 70 GPa and the corresponding value for steel is 210 GPa. If a total load P=75 kN is applied to the bar through a plate mounted on top of the bar and the lower end of the bar rests on a similar plate, calculate the normal stress in the mantle and in the core of the cylinder

Ans: $\sigma_{mantle} = 17.1\frac{N}{mm^2}, \sigma_{mantle} = 51.1\frac{N}{mm^2}$